<u>Quiz 2</u>

Course: Algebraic Topology II (KSM4E02) **Instructor:** Aritra Bhowmick

Time: 2:00PM – 4:00PM, 23rd April, 2026 **Total marks:** 20

Attempt any question. You can get maximum **15 marks**.

You may use the homology groups / cohomology rings (with $\mathbb{Z}$ or $\mathbb{Z}_2$ coefficients) of standard spaces like $S^n, \mathbb{RP}^n, \mathbb{T}^n$, etc without showing the computation. If you are using UCT or Künneth theorem, mention them clearly. If any $\mathrm{Tor}/\mathrm{Ext}$ module vanishes, mention with justification.

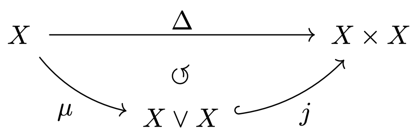
Q1. Given an R-module M, suppose $\mathrm{Tor}_1^R(M, N) = 0$ for any R-module N. Show that $\mathrm{Tor}_n^R(M, N) = 0$ for any R-module N, and for any $n \geq 2$. [4]

Q2. Suppose $n > m$. Show that there are no maps $\mathbb{RP}^n \to \mathbb{RP}^m$ inducing a nontrivial map in the 1^{st} cohomology with $\mathbb{Z}_2$-coefficients. [4]

Q3. Prove or disprove : $\mathbb{RP}^3$ is homotopy equivalent to $\mathbb{RP}^2 \vee S^3$. [4]

Q4. Assuming $k, l > 0$, show that there *does not* exist a map $f : S^{k+l} \to S^k \times S^l$ which induces a nontrivial map in the $(k+l)^{\text{th}}$-homology with $\mathbb{Z}$ coefficients. [4]

Q5. A based space X is called a *co-H space* if there is a map $\mu : X \to X \vee X$ such that the diagram commutes up to homotopy



Show that the cup product in the cohomology ring of X (with coefficients in any ring) is trivial in every positive degree. [4]