Quiz 1

Course: Algebraic Topology II (KSM4E02) **Instructor:** Aritra Bhowmick

Time: 2:00PM – 4:00PM, 22nd February, 2026 **Total marks:** 20

Attempt any question. You can get maximum **15 marks**.

Q1. Suppose $A \subset X$ is a *homotopy retract*, i.e, there is a map $r : X \to A$ such that $r \circ \iota \simeq \mathrm{Id}_A$. Show that $H_n(X) = H_n(A) \oplus H_n(X, A)$ for all n. [5]

Q2. A pointed space (X, x_0) is called *good* if there is an open neighborhood $x_0 \in U \subset X$ such that U strongly deformation retract onto x_0. Given good pointed spaces $(A, a_0), (B, b_0)$, consider the wedge $X = A \vee B$. Show that $H_n(X) = \tilde{H}_n(A) \oplus \tilde{H}_n(B)$ for all n. Show by example that the claim is not true if we do not consider reduced homology. $[4 + 1]$

Q3. Suppose $X = [0, 1]$ and $A = \{0\} \cup \left\{ \frac{1}{n} \mid n \geq 1 \right\} \subset X$. Show that $H_1(X, A) \ncong \tilde{H}_1(X/A)$ for the singular homology. [5]

Hint: You may use the fact that there exists a surjection $\pi_1(X/A) \twoheadrightarrow \prod_{i=0}^{\infty} \mathbb{Z}$.

Q4. Compute the singular homology groups of the following spaces. [5]

- X is the space obtained from S^2 by pinching two antipodal points.
- Y is the space obtained from S^2 by attaching an equatorial disc.
- Z is the space obtained from the torus by attaching a S^2 along the equator in the middle hole.
- U is the space obtained from $\mathbb{R}^3$ by removing the unit circle in the xy-plane with center at the origin.
- V is the space obtained from $S^3 = \mathbb{R}^3 \cup \{\infty\}$ by removing the unit circle in the xy-plane with center at the origin.