Mid-semester Examination

Course: Algebraic Topology II (KSM4E02) **Instructor:** Aritra Bhowmick

Time: 2:00PM – 5:00PM, 2nd March, 2026 **Total marks:** 30

Attempt any question. You can get maximum **25 marks**.

Q1. Compute the singular homology groups of $S^p \times S^q$ for $p, q \geq 0$. [5]

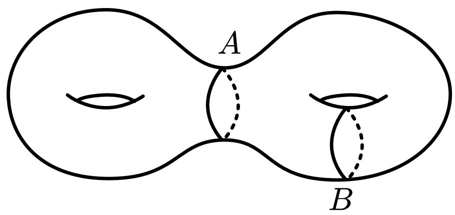
Q2. Compute the singular homology groups of $S^1 \times (S^1 \vee S^1)$. [5]

Q3. Compute the *relative* singular homology groups in the following cases. $[2 \times 4 = 8]$

a. $X = S^2$ and $A \subset X$ is a finite set.

b. $X = S^1 \times S^1$ and $A \subset X$ is a finite set.

c. X is the double torus, and A is the middle circle.



d. X is again the double torus, and B is the circle as above.

Q4. Prove *Brouwer fixed point theorem* : any continuous map $f : D^n \to D^n$ has a fixed point, where $D^n \subset \mathbb{R}^n$ is the closed unit disc. (**Hint :** Consider the *ray* joining $f(x)$ to x.) [4]

Q5. Show that any fixed-point free map $f : S^n \to S^n$ is homotopic to the antipode map. (**Hint** : For $x \in S^n$, can the convex sum of $f(x)$ and $-x$ pass through the origin?) [4]

Q6. Justify that on an *even* sphere the only nontrivial group acting freely is $\mathbb{Z}/2\mathbb{Z}$. [4]

Recall : A group G acts on a space X if there is a continuous map $\varphi : G \times X \to X$ such that $\varphi(e, x) = x$, and $\varphi(g, \varphi(h \cdot x)) = \varphi(g \cdot h, x)$ holds for $g, h \in G, x \in X$. The action is *free* if for any $g \neq e$ (where $e \in G$ is the identity), we have $\varphi_g : X \to X$ given as $x \mapsto g \cdot x$ is a fixed-point free map.