

Quiz 1

Course: Algebraic Topology I (KSM3E02)

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Time: 12:00PM – 1:00PM, 20th August, 2026

Total marks: 20

Name: _____

Marks: _____ /20

- A map $f : X \rightarrow Y$ is *null-homotopic* if f is homotopic to a constant map $X \rightarrow Y$.
- Given a map $f : X \rightarrow Y$, the *mapping cone* C_f is the quotient space

$$C_f := \frac{X \times [0, 1]}{(x, 0) \sim (x', 0), (x, 1) \sim f(x)}.$$

- Given a subspace $A \subset X$, a *retraction* is a map $r : X \rightarrow A$ such that $r|_A = \text{Id}_A$, i.e, $r \circ \iota = \text{Id}_A$ where $\iota : A \hookrightarrow X$ is the inclusion.
- A space X is *contractible* if it is homotopy equivalent to a point.

Answer **Q1** and one of **Q2 / Q3**.

Q1. Suppose $X = \mathbb{R}^3$ and A is one of the subspaces as below. Describe $X \setminus A$ as the wedge of circles and spheres up to homotopy equivalence. No need to provide explanations. [5 × 2 = 10]

- A is an infinite line.
- A is the union of n -many parallel infinite lines.
- A is the union of n -many infinite lines all passing through the origin.
- A is the unit circle S^1 centered at origin in the xy -plane.
- $A = \{(x, y, 0) \mid (x - 1)^2 + y^2 = 1\} \cup \{(x, y, 0) \mid (x + 1)^2 + y^2 = 1\}$.

Q2. Given a map $f : X \rightarrow Y$, show that the following are equivalent. [5 + 5 = 10]

- $f : X \rightarrow Y$ is null-homotopic.
- There exists a retraction $C_f \rightarrow Y$ from the mapping cone of f to the base X .

Q3. Given a space X , show that the following are equivalent. [5 + 5 = 10]

- X is contractible.
- Given any space Y , any map $Y \rightarrow X$ is null-homotopic.