Makeup Examination

Course : Topology (KSM1C03)

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17th January, 2026

Time: 09:30 - 12:30 **Total marks:** 65

Attempt **any** question. Maximum marks will be capped in accordance with the make-up examination rules and your continuous assessment marks.

Q1. Suppose X is a topological space. Show that the topology is indiscrete if and only if given any space Y, any function $f : Y \to X$ is continuous. $[5]$

Q2. A space X is called *scattered* if every nonempty subspace $Y \subset X$ contains an isolated point. $[3 + 2 = 5]$

a) Show that a scattered, T_1-space is totally disconnected.

b) Is it possible to remove the T_1 assumption?

Q3. Given $A, B \subset X$, prove the following. $[2 + 2 + 1 = 5]$

a) $\operatorname{int}(A \cap B) = \operatorname{int}(A) \cap \operatorname{int}(B)$.

b) $\operatorname{int}(A \cup B) \supset \operatorname{int}(A) \cup \operatorname{int}(B)$.

Given an example when $\operatorname{int}(A \cup B) \neq \operatorname{int}(A) \cup \operatorname{int}(B)$.

Q4. Suppose $f : X \to Y$ is a continuous surjection. If X is Lindelöf, show that Y is Lindelöf. $[5]$

Q5. Consider the space $X = \mathbb{R} \times \{0, 1\}$ with the dictionary order, and the induced order topology. $[3 + 2 = 5]$

a) Show that the subspace $Y = \mathbb{R} \times \{1\} \subset X$ is homeomorphic to the Sorgenfrey line $\mathbb{R}_\ell$ (i.e, $\mathbb{R}$ with the lower limit topology).

b) Conclude that $\mathbb{R}_\ell$ is T_5 (i.e, a completely normal T_1 space).

Q6. Let X be an uncountable space, equipped with the co-countable topology. For $A \subset X$, prove the following.

a) A is compact if and only if A is finite.

b) A is connected if and only if A is singleton or uncountable. $[(2 + 2) + (3 + 3) = 10]$

Q7. Show that any compact subset of the one-point compactification $\widehat{\mathbb{Q}} = \mathbb{Q} \cup \{\infty\}$ of $\mathbb{Q}$ is closed in $\hat{\mathbb{Q}}$. $[10]$

Hint: If x_n is a sequence of rational points converging to some rational number x, then note that $\{x\} \cup \{x_n \mid n \geq 1\}$ is a compact subset of $\mathbb{Q}$.

Q8. Consider the space $X = [0, 1] \cup 1^*$, where 1^* is a separate point. Consider the collection

$$\mathcal{B} := \{U \subset [0, 1] \mid U \text{ is open in the usual topology}\} \cup \{(a, 1) \cup \{1^*\} \mid 0 \leq a < 1\}.$$

a) Show that $\mathcal{B}$ is a basis for a topology on X, called the *telophase topology*.

b) On $[-1, 1]$ define an equivalence relation, : $x \sim y$ if and only if

$$x = y, \qquad \text{or} \qquad -1 < x, y < 1, \text{ and } x = -y.$$

Show that the quotient space $[-1, 1]/_\sim$ is homeomorphic to the telophase topology.

c) Give an example of two compact sets in the telophase topology, whose intersection is not compact. $[3+4+3 = 10]$

Q9. Recall, a space X is called *exhaustible by compacts* if there are compact sets $\{K_n\}_{n\geq 1}$ such that $X = \bigcup K_n$ and $K_n \subset \operatorname{int}(K_{n+1})$ for all $n \geq 1$. Suppose X is a T_2 space. Show the following are equivalent. $[5 + 5 = 10]$

a) X is exhaustible by compacts.

b) X is Lindelöf and locally compact.

Hint: Recall that finite union of compacts is compact, and interior of finite union contains the finite union of interiors.