Class Test 8

11th November, 2025

Name: _____

Time: 40 min **Marks:** ____/10

Q1. On $\mathbb{R}$, consider the topology $\mathcal{T}$ (known as the *discrete rational extension of* $\mathbb{R}$) generated by the basis

$$\mathcal{B} \coloneqq \{(a,b) \mid a,b \in \mathbb{R},\ a<b\} \cup \{\{q\} \mid q \in \mathbb{Q}\}\,.$$

Show that X is metrizable. [10]

Definitions/Hints

- Urysohn's metrization theorem : A second countable, T_3-space is metrizable.
- A space is T_3 if it is regular and T_0.
- A zero dimensional space is *completely regular*.
- A space is called *zero-dimensional* if there is a basis of clopen sets.