Class Test 7

4th November, 2025

Name: ____

Time: 40 min **Marks:** ____/10

Q1. On $\mathbb{R}^2$, consider $\mathcal{B}$ to be the collection of open discs (in the usual topology), with their horizontal diameters (except the center) removed, which is a basis for a topology, say, $\mathcal{T}$ on $\mathbb{R}^2$. Explicitly, a basic open set with center (x_0, y_0) and radius r is of the form

$$\left\{(x,y) \;\middle|\; (x-x_0)^2 + (y-y_0)^2 < r^2\right\} \setminus \left\{(x, y_0) \mid 0 < |x - x_0| < r\right\}.$$

The space $X = (\mathbb{R}^2, \mathcal{T})$ is called the *deleted diameter topology*. $[2 \times 5 = 10]$

a) Show that X is strictly finer than the usual topology on $\mathbb{R}^2$.

b) Show that X is functionally Hausdorff.

c) Show that X is not T_3.

d) Show that $Y = \mathbb{R} \times \{0\} \subset X$ is a closed discrete subspace.

e) Show that X is not Lindelöf.