Class Test 6

28th October, 2025

Name: _____

Time: 40 min **Marks:** ____/10

Q1. Given a space X, define an equivalence relation : $x \sim y$ if and only if they have the same open neighborhoods. Denote the quotient space as $\mathcal{K}(X)$ (known as the *Kolmogorov quotient* of X). $[4 + 2 + 4 = 10]$

a) Show that the quotient map $q : X \to \mathcal{K}(X)$ is both open and closed.

b) Show that $\mathcal{K}(X)$ is a T_0-space (i.e, a Kolmogorov space).

c) Show that X is regular if and only if $\mathcal{K}(X)$ is T_3.