Class Test 5

23rd October, 2025

Name: _____

Time: 40 min **Marks:** ____/10

Q1. Given any $A \subset X$, recall the boundary is defined as $\partial A \coloneqq \bar{A} \cap \overline{X \setminus A}$, and the subset A is called nowhere dense if $\mathrm{int}(\bar{A}) = \emptyset$. Prove the following. $[2 \times 5 = 10]$

a) Suppose U is open in X. Show that ∂U is nowhere dense.

b) Suppose C is closed in X. Show that ∂C is nowhere dense.

c) Give an example of some $A \subset X$, such that ∂A is not nowhere dense.

d) Suppose $A \subset X$ is nowhere dense, and closed. Show that $A = \partial U$ for some $U \subset X$ open.

e) Give an example of some nowhere dense $A \subset X$ such that A is not a boundary of any open subset of X.