Class Test 4

3rd October, 2025

Name: _____

Time: 30 min **Marks:** ____/10

Attempt **one** of the questions.

Q1. Let $f : X \to Y$ be a continuous map. Suppose,

- f is closed, i.e, for any closed set $C \subset X$, the image $f(C)$ is closed in Y, and
- f has compact fiber, i.e, for any $y \in Y$ the pre-image $f^{-1}(y)$ is compact in X.

Show that f is a proper map, i.e, for any compact set $K \subset Y$, show that the pre-image $f^{-1}(K)$ is compact in X. 10

Q2. Prove or disprove the following statements. $5 + 5 = 10$

a) Open subsets of a locally compact space is locally compact.

b) Closed subsets of a locally compact space is locally compact.