Class Test 2

2nd September, 2025

Name: _____

Time: 40 min **Marks:** ____/10

Q1. Given any space X, consider the equivalence relation : $x \sim y$ if and only if x and y are in the same connected component. Prove or give counterexample to **any 5** of the following statements.

a) $X/_\sim$ is a T_1 space.

b) $X/_\sim$ is a T_2 space.

c) If $X/_\sim$ is a discrete space, then X has finitely many connected components.

d) If $X/_\sim$ is an indiscrete space, then X is connected.

e) If $X/_\sim$ is a connected space, then X is connected.

f) If X is totally disconnected, then the quotient map $q : X \to X/_\sim$ is a homeomorphism.

$2 \times 6 = 12$