Class Test 1

26th August, 2025

Name: ____

Time: 40 min **Marks:** ____/10

Q1. Let $X = \mathbb{R}/\mathbb{Q}$ be the identification space, i.e, the quotient space induced by the relation $a \sim b$ if and only if $a, b \in \mathbb{Q}$ or $a = b \in \mathbb{R} \setminus \mathbb{Q}$. Let $q : \mathbb{R} \to X$ be the quotient map.

a) Describe the open sets $U \subset \mathbb{R}$ which are q-saturated (i.e, $U = q^{-1}(q(U))$).

b) What is the closure of the equivalence class $[x] \in X$ for any $x \in \mathbb{R} \setminus \mathbb{Q}$?

c) What is the closure of the equivalence class $[0] \in X$?

d) Determine (with brief explanation) whether X is T_2, T_1, or T_0. $1 + 2 + 2 + 1 = 6$

Q2. Let X be an infinite set, and fix a point $p \in X$. Consider the collection

$$\mathcal{T}_p := \{S \subset X \mid p \in S\} \cup \{\emptyset\}.$$

a) Verify that $\mathcal{T}_p$ is a topology on X (called the *particular point topology*).

b) Consider a sequence $\{x_n\}$ in X, whose tail (i.e, the subsequence $\{x_n\}_{n \geq N}$ for some $N \geq 1$) looks like

$$x, p, x, p, x, p, \ldots.$$

Show that x_n converges to x. If $x \neq p$, then show that the sequence does not converge to p.

c) Determine (with brief explanation) whether $(X, \mathcal{T}_p)$ is T_2, T_1, or T_0. $1 + 2 + 1 = 4$